



# Revisiting the 2009 Iranian Presidential Election



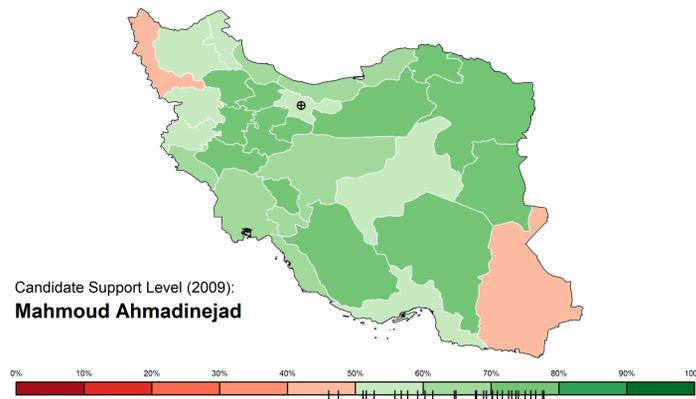
## Is there evidence of electoral unfairness?

### Abstract

In 2009, the Iranian people went to the polls to elect a president. The incumbent president, Mahmoud Ahmadinejad, officially received almost two-thirds of the votes cast. Reformist Mir-Hossein Mousavi was a distant second, followed by two lesser-known candidates, Mehdi Karroubi and Moshen Rezaee.

Almost immediately, protests broke out across Iran declaring the results of the vote fraudulent. However, the evidence for that fraud rested more on the presence of the protests than on concrete conclusions.

In 2009, the field of electoral forensics was not well-known, and its toolset was limited. In the intervening seven years, advances have been made in better understanding the relationships (or lack thereof) in a fair election. This research revisits the results of the Iranian 2009 presidential election, using a couple new testing methods.

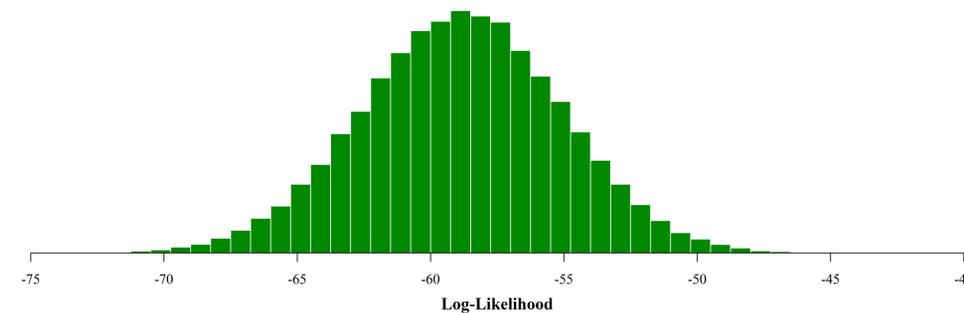


According to Forsberg (2014) the pmf is

$$\frac{\lfloor \theta \rfloor}{\theta} \cdot \log \left( \frac{d+1}{d} \right) + \begin{cases} 0 & 10^{\theta - \lfloor \theta \rfloor} < d \\ \frac{1}{\theta} \left( \theta - \lfloor \theta \rfloor + \log \left( \frac{1}{d} \right) \right) & d \leq 10^{\theta - \lfloor \theta \rfloor} < d + 1 \\ \frac{1}{\theta} \log \left( \frac{d+1}{d} \right) & d + 1 \leq 10^{\theta - \lfloor \theta \rfloor} \end{cases}$$

The generalized Benford distribution is superior to the usual Benford distribution because it has all of the limiting properties of the Benford distribution, and it allows for different upper bounds,  $10^\theta$ .

As the division sizes,  $10^\theta$  differ from division to division, a simple goodness-of-fit test cannot be used. I simulated the distribution of the log-likelihood for the given turnouts and the generalized Benford distribution and compared the observed log-likelihood to that distribution. The following is the observed distribution of log-likelihoods for the turnout figures.



From this, we can estimate the 95% confidence interval for log-likelihoods is from -66.3 to -51.6. The log-likelihoods for the four candidates are -58.9 for Ahmadinejad, -64.3 for Mousavi, -56.0 for Karroubi, and -60.7 for Rezaee. All are within the confidence interval.

This test offers no evidence of unfairness/fraud in the 2009 Iranian presidential election.

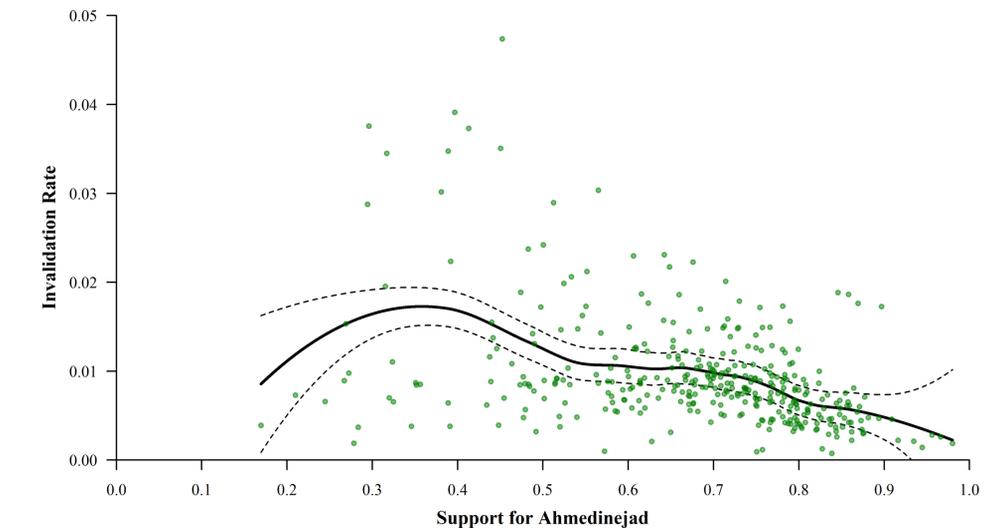
### Differential Invalidation

Benford's laws assume that free and fair elections follow a specific distribution without offering logical evidence (Deckert et al. 2011). Another test looks at a consequence of an election being "fair" — that **the probability of a ballot being counted is the same, regardless of who it is cast for.**

$$H_0 : \beta = 0$$

In other words, if there is a relationship between the invalidation rate and the support rate for a candidate, there is evidence of **differential invalidation**. As the dependent variable is a proportion (invalidation rate), one should use a binomial-based generalized linear model estimated using maximum quasi-likelihood estimation.

For the 2009 election, the following is the invalidation plot for Ahmadinejad with a lowest curve and approximate 95% confidence interval. Note that a horizontal line does not fit between the bounds. This is graphical evidence of a significant relationship between the two variables.



The regression model also indicated a significant — and negative — relationship between the two variables ( $p=0.0004$ ). This provides strong evidence of differential invalidation. That is, we have evidence that the election was unfairly counted in favor of incumbent president Ahmadinejad.

### References

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### For Further Information

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This poster, some supplemental materials, and an extensive article can be accessed at

<http://www.electoralforensics.org/commentary/?p=5239>

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